J.K. SHAH CLASSES

MATHEMATICS & STATISTICS

FYJC TEST

DURATION – 2 HR MARKS – 40

Q1. Attempt ALL FOUR of the following 2 MARKS EACH (08)

01. A(3,5) and B(4,1) . Find the locus of point P such that $l(AP)^2 + l(BP)^2 = 60$

SOLUTION :

let P(x,y) be any point on the locus , A(3, 5) ; B(4,1)

As per the given condition

$l(AP)^2 + l(BP)^2 = 60$

$(x - 3)^2 + (y - 5)^2 + (x - 4)^2 + (y - 1)^2 = 60$

$x^2 - 6x + 9 + y^2 - 10y + 25$

$+ \quad x^2 - 8x + 16 + y^2 - 2y + 1 = 60$

$2x^2 - 14x + 25 + 2y^2 - 12y + 26 = 60$

$2x^2 + 2y^2 - 14x - 12y + 51 - 60 = 0$

$2x^2 + 2y^2 - 14x - 12y - 9 = 0$ ……. Locus of P

02. find the equation of the locus of the point which is equidistant from the points A(–5, 2) and B(4,1) .

SOLUTION :

let P(x,y) be any point on the locus , A(–5,2) ; B(4,1)

As per the given condition

$PA = PB$

$PA^2 = PB^2$

$(x + 5)^2 + (y - 2)^2 = (x - 4)^2 + (y - 1)^2$

$x^2 + 10x + 25 + y^2 - 4y + 4 = x^2 - 8x + 16 + y^2 - 2y + 1$

$10x - 4y + 29 = -8x - 2y + 17$

$18x - 2y + 12 = 0$

$9x - y + 6 = 0$ ………… Equation of the locus

03. if the origin is shifted to the point $(-2,1)$, axes remaining parallel to original axes , if the new coordinates of point A are $(7,-4)$, find the old coordinates of point A

SOLUTION

Origin shifted to $(h,k) \equiv (-2,1)$

New coordinates $(X,Y) \equiv (7,-4)$

$X = x - h \qquad Y = y - k$

$7 = x + 2 \qquad -4 = y - 1$

$x = 5 \qquad y = -3$

old coordinates $(x,y) \equiv (5,-3)$

04. the point $(3,8)$ becomes $(-2,1)$ after shift of origin . Find the coordinates of the point , where the origin is shifted

SOLUTION

origin shifted to (h,k)

old coordinates $(x,y) \equiv (3,8)$

new coordinates $(X,Y) \equiv (-2,1)$

$X = x - h \qquad Y = y - k$

$-2 = 3 - h \qquad 1 = 8 - k$

$h = 5 \qquad k = 7$

origin is shifted to $(5,7)$

Q2. Attempt ANY TWO of the following 4 MARKS EACH (08)

01. find equation of the locus of the point P such that join of $(-2,3)$ and $(6,-7)$ subtends right angle at P

SOLUTION :

let $P(x,y)$ be any point on the locus , $A(-2,3)$; $B(6,-7)$

$$PA^2 + PB^2 = AB^2$$

$$\left[(x+2)^2+(y-3)^2\right] + \left[(x-6)^2+(y+7)^2\right] = (-2-6)^2 + (3+7)^2$$

$$x^2 + 4x + 4 + y^2 - 6y + 9 + x^2 - 12x + 36 + y^2 + 14y + 49 = 64 + 100$$

$$2x^2 + 2y^2 - 8x + 8y + 13 + 85 = 164$$

$$2x^2 + 2y^2 - 8x + 8y + 98 - 164 = 0$$

$$2x^2 + 2y^2 - 8x + 8y - 66 = 0$$

$x^2 + y^2 - 4x + 4y - 33 = 0$ ………. equation of the locus

02. find the equation of the locus of the point whose distance from (−2,1) is thrice its distance from (1,4)

SOLUTION :

let P(x,y) be any point on the locus , A(−2,1) ; B(1,4)

As per the given condition

$PA = 3PB$

$PA^2 = 9PB^2$

$(x + 2)^2 + (y - 1)^2 = 9\left[(x - 1)^2 + (y - 4)^2\right]$

$x^2 + 4x + 4 + y^2 - 2y + 1 = 9(x^2 - 2x + 1 + y^2 - 8y + 16)$

$x^2 + y^2 + 4x - 2y + 5 = 9(x^2 + y^2 - 2x - 8y + 17)$

$x^2 + y^2 + 4x - 2y + 5 = 9x^2 + 9y^2 - 18x - 72y + 153$

$0 = 8x^2 + 8y^2 - 22x - 70y + 148$

$8x^2 + 8y^2 - 22x - 70y + 148 = 0$

$\div 2$

$4x^2 + 4y^2 - 11x - 35y + 74 = 0$ ……… equation of the locus

03. the equation of the locus is $x^2 - 4x - 6y - 20 = 0$. If the origin is shifted to the point (2,−4) , axes remaining parallel , find the new equation of the locus

SOLUTION

origin shifted to $(h,k) \equiv (2,-4)$

old coordinates $\equiv (x,y)$, new coordinates $\equiv (X,Y)$

$X = x - h$; $Y = y - k$

$X = x - 2$; $Y = y + 4$

$x = X + 2$; $y = Y - 4$

OLD equation of the locus : $x^2 - 4x - 6y - 20 = 0$

NEW equation of the locus : $(X + 2)^2 - 4(X + 2) - 6(Y - 4) - 20 = 0$

$X^2 + 4X + 4 - 4X - 8 - 6Y + 24 - 20 = 0$

$X^2 - 6Y = 0$

Q3. Attempt ANY FOUR of the following — 3 MARKS EACH (12)

01. Find the value of k for which points P(1,−2) , Q(3,1) and R(5,k) are collinear

$$m_{PQ} = \frac{1+2}{3-1} = \frac{3}{2}$$

$$m_{QR} = \frac{k-1}{5-3} = \frac{k-1}{2}$$

Since P ,Q ,R are collinear ,

$m_{PQ} = m_{QR}$.

$$\frac{3}{2} = \frac{k-1}{2}$$

$$3 = k - 1$$

$$\therefore k = 4$$

02. find line passing through (2,5) and parallel to 3x − 4y − 7 = 0

$3x - 4y - 7 = 0$

$$m = \frac{-a}{b} = -\frac{3}{-4} = \frac{3}{4}$$

Required Line

$m = 3/4$, passing through (2,5)

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{3}{4}(x - 2)$$

$$4y - 20 = 3x - 6$$

$$3x - 4y + 14 = 0$$

03. find line whose x − intercept is 3 and which is perpendicular to line 3x−y+ 23= 0

$3x - y + 23 = 0$

$$m = \frac{-a}{b} = -\frac{3}{-1} = 3$$

Required Line

$m = {}^{-1}/_{3}$, passing through (3,0)

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{-1}{3}(x - 3)$$

$$3y = -x + 3$$

$$x + 3y - 3 = 0$$

04. find the measure of acute angle between the lines 12x − 4y = 5 & 4x + 2y = 7

$12x - 4y = 5$

$$m_1 = -\frac{a}{b} = -\frac{12}{-4} = 3$$

$4x + 2y = 7$

$$m_2 = -\frac{a}{b} = -\frac{4}{2} = -2$$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1.m_2}\right|$$

$$= \left|\frac{3-(-2)}{1+3(-2)}\right|$$

$$= \left|\frac{5}{-5}\right|$$

$$= 1$$

$$\theta = 45^\circ$$

05.

find distance between $6x + 8y + 21 = 0$ &

$3x + 4y + 7 = 0$

Line 1 : $6x + 8y + 21 = 0$

Line 2 : $\cancel{3x + 4y + 7 = 0}$

$6x + 8y + 14 = 0$

Distance between the two parallel lines :

$$d = \left| \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right|$$

$$= \left| \frac{21 - 14}{\sqrt{6^2 + 8^2}} \right|$$

$$= \frac{7}{10}$$

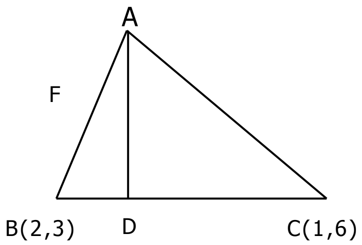
$$= 0.7 \text{ units}$$

Q4. Attempt ANY THREE of the following 4 MARKS EACH (12)

01.

ΔABC . A(1,4) , B(2,3) , C(1,6) .

Find equations of altitudes AD , median BE



ALTITUDE AD

$$m_{BC} = \frac{6-3}{1-2} = -3$$

$$\therefore \; m_{AD} = {}^{1}/_{3} \qquad (AD \perp BC)$$

$m = {}^{1}/_{3}$, A(1,4)

$$y - y_1 = m\,(x - x_1)$$

$$y - 4 = \frac{1}{3}(x-1)$$

$$3y - 12 = x - 1$$

$$x - 3y + 11 = 0$$

MEDIAN BE

$$E \equiv \left(\frac{1+1}{2}, \frac{4+6}{2}\right) \equiv (1,5)$$

B(2,3) , E(1,5)

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 3 = \frac{5-3}{1-2}(x-2)$$

$$y - 3 = -2(x-2)$$

$$2x + y - 7 = 0$$

02. Find equation of line which passes thro' point of intersection of lines $x+2y-3=0$ and $3x+4y-5=0$ and which is perpendicular to the line $x-3y+5=0$

Point of Intersection

$$x + 2y = 3 \quad \times\; 2$$

$$3x + 4y = 5$$

$$3x + 4y = 5$$

$$2x + 4y = 6$$

$$x \qquad = -1$$

subs in (1) $y = 2$ $\therefore (-1,2)$

$$x - 3y + 5 = 0$$

$$m = \frac{-a}{b} = \frac{-1}{-3} = \frac{1}{3}$$

Equation of required line

$m = -3$, passing through $(-1,2)$

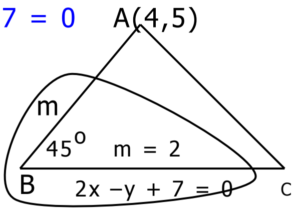
$$y - y_1 = m\,(x - x_1)$$

$$y - 2 = -3(x+1)$$

$$3x + y + 1 = 0$$

03.

Find equations of the lines passing through the point (4,5) and making an angle of 45° with the line $2x - y + 7 = 0$



STEP 1 :

$2x - y + 7 = 0$

$m = \frac{-a}{b} = \frac{-2}{(-1)} = 2$

STEP 2 :

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 . m_2}\right|$$

$$\tan 45 = \left|\frac{m - 2}{1 + m(2)}\right|$$

$$1 = \left|\frac{m - 2}{1 + 2m}\right|$$

$\frac{m - 2}{1 + 2m} = 1$

$m - 2 = 1 + 2m$

$m - 2m = 1 + 2$

$-m = 3$

$m = -3$

$\frac{m - 2}{1 + 2m} = -1$

$m - 2 = -1 - 2m$

$m + 2m = -1 + 2$

$3m = 2$

$m = \frac{1}{3}$

STEP 3

Equation of AB : $m = -3$, A(4, 5)

$y - y_1 = m(x - x_1)$

$y - 5 = -3(x - 4)$

$y - 5 = -3x + 12$

$3x + y - 17 = 0$

Equation of AC : $m = \frac{1}{3}$, A(4, 5)

$y - y_1 = m(x - x_1)$

$y - 5 = \frac{1}{3}(x - 4)$

$3y - 15 = x - 4$

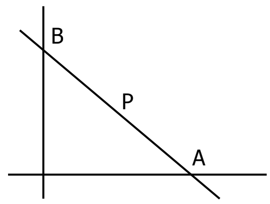
$x - 3y + 15 - 4 = 0$

$x - 3y + 11 = 0$

04.

Find the equation of line which passes through (1,2) and the midpoint of the portion of the line $3x - 4y + 24 = 0$ intercepted between the coordinate axes .

SOLUTION



STEP 1 :

$3x - 4y + 24 = 0$

put $y = 0$; $3x + 24 = 0$

$3x = -24$ $\therefore$ A $(-8,0)$

put $x = 0$; $-4y + 24 = 0$

$-4y = -24$ $\therefore$ B (0.6)

STEP 2 :

P be the midpoint of AB

Using midpoint formula

$$P \equiv \left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2}\right)$$

$$\equiv \left(\frac{-8 + 0}{2} , \frac{0 + 6}{2}\right)$$

$P \equiv (-4,3)$

STEP 3 :

Line is passing through (1,2) & (−4,3)

Equation of the line

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 2 = \frac{3 - 2}{-4 - 1}(x - 1)$$

$$y - 2 = \frac{1(x - 1)}{-5}$$

$-5y + 10 = x - 1$

$x + 5y - 11 = 0$